Support for Teaching Mathematics of the Blind by Sighted Tutors Through Multisensual Access to Formulas With Braille Converters and Speech

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ABSTRACT

Nowadays, teaching various subjects at school is successfully supported by information and remote technologies such as Google Class, Moodle and others. Nevertheless, students with special needs such as the visually impaired (BVI) face incredible barriers to using such remote technologies, especially with learning mathematics or physics. The main problem is that BVI uses different tools and techniques than their sighted peers, i.e., a different way of working with mathematical expressions or a lack of the possibility to edit graphics. Traditional methods such as the Brailler, figure models or cubarithms are still used. Another challenge is that there are entirely different systems of presenting formulas in different countries, so-called Braille mathematical notations. To overcome these barriers, we propose universal tools to assist sighted teachers and BVI students in remote training math using a multimodal form of editing of mathematical formulas. It consists of the simultaneous combination of three forms of presentation of math formulas in graphical form for the teacher, intelligent reading through speech synthesis and Braille mathematical notation for BVI. It is possible thanks to the use of intelligent converters between formats such as MathML, intelligent text and Braille and dedicated editors that allow for creating math documents by students and teachers.

CCS CONCEPTS

• **Information systems** → *Synchronous editors*; • **Human-centered computing** → **Accessibility systems and tools**; **Interactive systems and tools**; • **Applied computing** → **Interactive learning environments**.

KEYWORDS

blind students, teaching math, Braille math notation, multimodal access, math formula converters

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1 INTRODUCTION

Teaching most of the subjects in school today is successfully supported by information and remote technologies such as Google Class, Moodle platform and others. It also applies to sighted teachers and visually impaired students (BVI). Although there are various electronic support solutions in areas related to teaching languages and subjects such as history or geography, the matter is much more difficult in mathematics, physics or chemistry. The main problem is that BVI use entirely different tools, methods and techniques to learn mathematics than their sighted peers. For example, they have to perform a large part of the calculations in memory solving arithmetic or algebraic problems. They also use a completely different way of writing and reading mathematical formulas. Expressions are entered by them in a linear form and then read in the Braille alphabet or using synthetic speech. It makes working with expressions very slow and requires a lot of concentration from the student and the teacher. Working with graphics is also very difficult because the appearance of a graph or figure, even if the student has a convex drawing at his disposal, must create a snapshot of this graph in his mind. For these reasons, in many countries, traditional solutions such as the Brailler, figure models or cubarithms [22] are still used. Therefore there is a great need for computer-based and remote support for the math teaching process of BVI. Research on these issues has been conducted for many years, but there is a lack of sufficiently complete and entirely accepted solutions. The matter is further complicated because different countries use different *Braille notations*

Therefore, this paper presents our attempts at tools to assist BVI and sighted tutors in learning and teaching math. The rest of the paper is structured as follows: First, we present the existing BVI math support solutions available in different countries. Among them, various Braille mathematical notations and tools for their conversion play a crucial role. Then we will present the research we conducted among teachers and students as part of two ongoing projects. In the next section, we present our approach to the multimodal presentation of mathematical formulas in three simultaneous forms for the teacher in graphic form and for the blind student in the form of intelligent speech reading and Braille notation in Polish and English. Such presentation is possible thanks to converters that convert formulas between MathML Braille and semantic speech formats. The next section will present the solutions developed using

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conversion methods. Then we present how the developed applications have been tested and used by teachers and students. We will summarize with conclusions and plans for the future

2 CURRENTLY AVAILABLE MATH TEACHING OPPORTUNITIES FOR BLIND STUDENTS

According to our research conducted in 2015, 2020 [20], most BVI use traditional tools to learn mathematics. These are cubarithms [22] for solving tasks with columnar layout operations or convex drawings [28] or figure models for teaching geometry. However, there have been some attempts to electronic assist the blind in learning mathematics. Unfortunately, most of them are incomplete or unusable for some countries. However, we will introduce them in this section and outline the related problems.

2.1 Braille notations

Immediately after the creation of the louis Braille alphabet in 1824 [17] people began to think about writing mathematical expressions in this system as well. Unfortunately, work on these issues was carried out in different countries simultaneously. As a result, nowadays, we are dealing with many Braille mathematical notations, each of which is used in a different country. This situation causes the biggest problem in which blind people from different countries cannot communicate in the field of mathematics. Additionally, with the spread of computer technologies and devices such as Braille displays and printers, Braille notations were created exclusively for electronic devices. The most commonly used Braille math notations in different countries are as follows:

- Nemeth code USA [25] developed in 1946 - 1974 by Dr Abraham Nemeth, Braille Authority of North America.
- British notation [26] developed in 1970 - 2005 by the Braille Authority of the United Kingdom.
- Unified English Braille (UEB) [2] developed in1987 - 1993 by Dr T. V. Cranmer and Dr A. Nemeth. International Council on English Braille.
- Marburg code (Germany) [12] developed in1955 - 1992 by dr. Epheser Deutsche Blindenstudienanstalt of Marburg.
- Polish Braille Math notation BNM [8] developed in 2002 - 2011 by the Polish Association of the blind.
- Mathematique Braille code (France) [33] developed in 2001 - 2007 by Louis-Auguste Antoine. Evolution du Braille Francais.
- Italian Braille code [15] developed in1998 - 2003 by the "Regina Margherita" Library for the Blind.

Other countries such as Russia, Portuguese and Japan also have their notations [9].

In our solutions, we used two Braille notations, namely the *English universal Braille Notation UEB* and the *Polish mathematical Braille notation (BNM)*. The *BNM* notation was developed as a modification of the *Marburg* used in Germany and has many symbols derived from it. We must emphasize here that *UEB* and *BNM* are the most contextual writing systems. It means that the meaning of a given symbol depends not only on its shape, i.e. the Braille points it is made of but also on what other symbols are present in its immediate vicinity. This phenomenon causes those conversion formulas from these systems to other formats by computer programs to be not an easy task.

Another very popular notation in the world is the *NemethCode* [25] used in the US. While *UEB, Marburg* and *BNM* concisely encode mathematical expressions, the idea behind *Nemeth* was to make the Braille notation similar to the appearance of mathematical writing used by seers. Therefore, the formulas that are written in *Nemeth Code* take up much space because many mathematical symbols must be encoded with two or more characters.

To better explain the essence of differences in these notations, We show a small demonstrative example. Let us take the following formula:

$$\frac{\frac{1}{2}+\frac{1}{3}}{6}$$

Its expression in the *Nemeth code* is shown in the figure 1. In turn, the same formula in the Polish mathematical notation is shown in the figure 2. As we can notice, even from this simple example, the differences in the concepts of both notations are significant. There are symbols in each notation, i.e. the beginning of the fraction, the end of the fraction, the fraction bar, and others. However, they have a different arrangement of Braille points. We can notice that in the Polish notation based on *Marburg*, the brevity of the notation is emphasized, which means that certain symbols may be omitted in some contexts. On the other hand, a formula in *Nemeth code* takes much more place because it has more characters. It is also easier to process automatically because each part of the formula has beginning and end tags, and the occurrence of symbols does not depend on their context. These differences in mathematical notations cause significant problems inconvenient communication between students and teachers from different countries. Therefore, there is a need to create universal solutions to support this communication. Taking it into account, we created such solutions for students from Poland, Ireland and the Netherlands

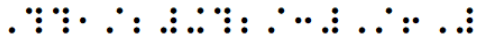


Figure 1: The formula in Nemeth code

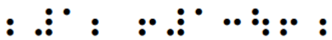


Figure 2: The formula in Polish Braille Math Notation

2.2 Other no-Braille notations

In addition to mathematical Braille notations, several systems can be used by the blind, although they are not strictly intended for them. A good example may be *AsciiMath* notation [5]. It is a simple code that allows writing math expressions using ASCII characters available directly from the computer keyboard. Its use is additionally facilitated by the fact that converters allow converting formulas from AsciiMateh to MathML and vice versa.

Another way of writing mathematical expressions that can be useful for the blind is the *LaTex* language and system [18]. Since the source text of a LaTex document is a linear notation, there have long been ideas for the blind to use it. A blind person can quickly write and read the source code of a mathematical document in the *LaTex* language. Then a document can be processed into a graphic form, e.g. to a pdf file, which is readable for a sighted person, and into a Braille form by an appropriate program. However, since the *LaTex* language covers all math and has many commands, learning them can be difficult for blind students, especially in early primary school. More over, not many teachers know this language yet. Because of the length and complexity of the LaTex commands, it is very problematic to write a math document fast and faultlessly, e.g. during the lesson in the classroom. To facilitate the work with *LaTex*, several programs that convert source documents into spoken or Braille forms have been created. One such solution is *MathTalk* [30]. Although the project was continued, it stopped being supported in 2021. Another solution is *Aster* (Audio System for Technical Readings) [27]. In Austria, at the University of Linz, a *LaBraDoor* system was developed [23]. The *LaBraDoor* acronym states for LaTex to Braille Door. It is a fully functional tool that allows Braille textbooks to contain mathematical content such as formulas, tables and graphs. The source file for such a handbook is a *LaTex* document. The conversion is carried out in several stages using the universal Braille conversion library (UMCL) [3]. The mathematical formulas in the resulting document are presented in *Marburg* notation. Another complete solution was a *BraMaNet* software developed by the Handicap section of the Lyon 1 University (France) [4] where the mathematical formulas are presented in French Braille notation.

The first Polish solution inspired by the *LaTex* language was the *Translator* software. It was a program that converts a LaTex document into a whole Braille handbook with such elements as mathematical expressions, tables, footnotes, and others. The *Translator* supported the Polish *BNM* notation in a slightly extended version. The continuation of the *Translator* project is the *Euler Science* software [13]. It is an extensive editor for creating mathematical textbooks in which *LaTex* is an intermediate form of a mathematical document.

Despite the attempts mentioned above, more traditional tools are still used to create textbooks that do not support the automated creation of mathematical formulas. These are applications such as *Winbraille* [6] or *Duxburry* [24, 32] as well as *Tiger SOFTWARE SUITE* [31]. The preparation of a mathematical text using these solutions in many steps is done manually. At first, a scanned pdf or paper textbook is then subjected to the OCR text recognition process. Then the text is manually corrected, and the math formulas are deleted. Instead, the editor enters them manually, sequentially entering the appropriate characters of the formula in the selected Braille notation. This operation can be done using a specially simulated keyboard in which the Braille dots correspond to the f, d, s, j, k, l keys on the standard QWERTY keyboard. As we can say, this method is time-consuming and requires much work.

2.3 Other applications and solutions

In addition to the solutions that can be used for preparing math textbooks, several programs support the teaching itself. One of the most exciting solutions of this kind is *Desmos* [11]. It is a web application that assists BVI in drawing mathematical graphs and their exploration. The student can enter the formula describing the function, i.e. quadratic equation, and the program generates its graph in a browser. Then, the graph is played as a specially generated audio signal. In this auditory representation, higher pitch signals indicate larger Y values of the graph, and analogically, lower pitch signals indicate fewer y values. The graph is played from left to right, so time reflects the X coordinates of the graph. In this way, a blind user can create an audial snapshot of a function graph, e.g. a parabola or a line in his mind. A *Desmos* system is available in several languages, and in addition to the tool mentioned above, it offers several more applications, such as a scientist calculator or figure graph editor.

Another complex solution is the *Lambda* application [29], *LAMBDA* acronym states for Linear Access to Mathematics for Braille Device and Audio-synthesis. It is an extensive application where the blind can create a mathematical document independently. She/He can perform any operations on formulas, insert symbols from available lists and edit math expressions. The *Lambda* system authors introduced their Braille notation based on characters consisting of not 6 but 8 points. As a result, unlike in traditional Braille notations, in *Lambda* many math symbols can be written with one character. In addition to displaying formulas in Braille, they are read using synthetic speech in a screen-reading program used by the blind, such as JAWS or NVDA. The system is available in German, Portuguese, Italian, and English. The only barrier against its spread in different countries is that users who would like to use it should learn the new Braille notation developed in this system. It is used extensively in Italy and several other countries.

Another obvious solution that a blind student can use is the equation editor in MS Word. It is accessible from the keyboard level, which means that a student can write the equations independently. However, there are problems reading them with a screen reader program like JAWS or NVDA. There are appropriate plugins for screen readers, but they work in English, which is not convenient for students from other countries, such as Poland.

2.4 Braille Math conversion tools

Since there are many Braille mathematical notations, there is a need to convert formulas from graphical representation of sighted to Braille form and vice versa. There are several approaches and attempts to fix this problem. One of them is the *MathInBraille* tool that is presented in the work entitled "MathInBraille online converter" [16]. It is a web portal that allows for the automatic conversion of electronic mathematical documents (containing formulas) into German *Marburg* Braille notation. Such formulas can also be read through speech in German. This tool is an open service that can be used for free.

Another commonly known tool is the web application *RoboBraille* [10], which enables fully automatic conversion of text to one of the alternative formats - including *Nemeth*. This website was developed at Stanford University. It allows students who use this notation to convert educational materials to Braille format on their own. *Robobrajl* is available in English.

Another approach is the *UMCL* converter [3], which tried to solve the problem of the lack of unified mathematical Braille notation in a very flexible way. *UMCL* is an open-source library that includes converters for various Braille math notations. It supports conversion between mathematical Braille notations: French, Italian, *Marburg, Nemeth, UEB,* and *MathML* and it LaTeX. As one of the few, this tool allows converting from LaTex and MathMl to Braille and reverse conversion (i.e. from mathematical Braille notation to MathMl or LaTex. This library can be used in standard way or as a remotely accessible web service. *UMCL* also allows converting documents from one Braille mathematical notation to another. Due to this, it allows blind users of different nationalities to exchange technical and mathematical documents.

In addition to the *UMCL* technology, there are also various converters for national Braille notations. An example of such a solution can be a converter between MathML, and the Spanish Braille mathematical notation [1]. The system was designed as a portable programming library whose task was to resolve ambiguities in Spanish mathematical Braille.

However, the most extensive and universal converter of mathematical formulas between Braille and MathML and LaTex formats is the *Liblouis* [19] library. Its universality is confirmed because it is widely used in many programs for creating Braille documents, software for Braille printers, and screen readers. Because it is the technology we used in our solutions, we will present it in more detail in one of the following sections.

3 OUR INVESTIGATIONS AMONG TEACHERS AND STUDENTS

In order to investigate the scale of the problem and the needs concerning teaching mathematics to blind and partially sighted, we conducted our first preliminary research in 2014. Our research focused on the computerization of the teaching process in the subject of mathematics in schools where blind students attend. They were also trying to identify the needs of respondents in this matter. The research was conducted among math teachers, blind and visually impaired students, and parents from mainstream and specialized schools. The research methods were electronic surveys and interviews conducted through telephone calls. One hundred forty-two respondents were recruited for the study, 104 completed the electronic questionnaire, and 38 were interviewed by phone. The questions included in the survey and interviews were divided into three thematic groups: ways of learning mathematics, the use of information technology in teaching mathematics, and demand for functions supporting the mathematical education of pupils with visual impairment. The teachers pointed to the big problems students face with tasks related to fractions, percentages, and geometric figures, particularly with imagining the mutual dependence of spatial figures. They also confirmed visually impaired pupils' much slower pace of work than the speed of work of sighted peers. According to parents' opinions (28%), the most incredible difficulty in learning new mathematical problems is caused by the lack of appropriate help and limited imagination concerning more complicated mathematical relationships. A significant disadvantage is a large volume and weight of the Braille book and the fact that mathematical aids are difficult to access, expensive and often appear in the English language version. Parents also pointed to the lack of appropriate software for speech synthesis and a deficit of educational games. They also emphasized the existence of a non-uniform notation for mathematical expressions on different devices. They said that a mathematical expression written on one device could not be processed into correct Polish Braille mathematical notation or a graphic form for a sighted person. All these difficulties cause the matriculation examination results in mathematics, especially for blind students, to be worse than the results of their sighted colleagues.

The research also revealed that many teachers (48%) do not know Braille. They are mainly teachers from mainstream schools. Moreover, teachers who know the Braille alphabet works mainly in special schools and are also familiar with *BNM*. In the case of students, 47% (only partially sighted) do not know Braille. The rest (53%, (blind and partially sighted) know them on a different level. The complete knowledge of *BNM* was declared by 41% of students from the group completely blind. More over, the research showed a high level of general computerization among the teachers (71%), parents (73%) and students (about 64%). However, the computer is not used to learn mathematics because there are no appropriate tools, according to respondents' opinions. For example, only 14% of students search the Internet for materials and tools helpful in learning mathematics. Teachers, especially those in mainstream schools (91%), are ready to use new, friendly technologies to help them teach mathematics.

This review shows that teachers and parents are very familiar with computer technology and their opening to new technological solutions. At the same time, the low-level use of technology in mathematics teaching of BVI indicates a great need in this regard. This need is additionally confirmed by the lack of knowledge of Braille by teachers who teach blind students and parents of BVI. Therefore, our research also included a few open questions about the solutions teachers would like to see in potential future supporting platforms.

4 OUR SOLUTIONS FOR LEARNING AND TEACHING MATH

Addressing the problems related to mathematical communication between sighted teachers and blind students and our preliminary findings, we have developed several solutions to facilitate such communication. We have undertaken an educational platform supporting mathematics teaching to visually impaired and blind students as a first challenge. It was a system consisting of five desktop applications running under the control of the Windows operating system. This set of applications was completed with a web portal containing pieces of information and educational materials for teachers.

After gathering experience using this first solution, the research was continued, albeit in a modified and extended form. A new web-based system for learning math for BVI students was developed. A platform was realized as a project carried out with the participation of partners from 3 countries, namely Poland, the Netherlands and Ireland.

Our last idea is an application designed for teachers to familiarize themselves with Polish Braille mathematical notation. All these solutions use the method of converting mathematical formulas between graphical form, semantic spoken representation, Braille notations, and converters based on the Liblouis library. We will present them in detail in the following sections, emphasizing the multimedia presentation of mathematical expressions and the possibility of operating on mathematical formulas.

4.1 Conversion of math formulas

As we have already mentioned, the blind use linear representations of formulas, unlike the sighted, who use two-dimensional graphical structures. In order to ensure adequate math communication, there is a need for conversion between different representations of math expression that should be made on-the-fly during student and teacher works with math content. The two-dimensional representation of a formula can be saved electronically, for example, in the form of equations in Microsoft Word or another program, or presented in a web browser by saving it in the MathMl format. The linear representation can be one of the notations such as Braille, AsciiMath or LaTex described in the previous sections, or a text form that a speech synthesizer can semantically read.

Our idea to facilitate such mathematical communication is about presenting mathematical expressions in a multimodal and multimedia manner. To do this, we use several formats simultaneously in one application dedicated to students and teachers. For example, the expression that the sighted teacher can view graphically is presented in the web browser, thanks to the fact that it is written in Hypertext in MathML. After the blind user indicates it with the keyboard, the same expression should be presented to her/him in a few linear forms. First, it is displayed on the Braille display in the form of a point, considering the Braille mathematical notation the student uses. Second, at the same time, this formula is read for the student via a speech synthesizer as a semantic text, taking into account its structure and particular math operators, variables and other elements. This semantic uses a mathematical language similar to what a teacher would use when reading this formula in nature. So formula elements such as the beginning of the fraction, fraction bar, indexes, and others should be read in proper order, and good mathematical language practise. In addition to showing the formulas, the teacher and the student should be able to edit them conveniently for each one. This way of multimodal simultaneous interactive presentation is the key challenge that should be ensured to work quickly and conveniently with the mathematical content during the lesson or in remote flipped class teaching mode. Converters are needed to make interactive communication possible to process one formula on the fly between different representations in different situations. Therefore, in our solutions, we applied the following converters:

(1) *MathML to UEB* english Braille Converter
(2) *UEB to MathMl* converter
(3) *MathML to semantic Polish or English text* converter,
(4) *AsciiMath to MathMl* converter
(5) *MathMl to AsciiMath* converter.
(6) *MathML to Polish Braille BNM* converter,
(7) *BNM to MathMl* converter,

We must emphasize that all these converters work in the background while the student and teacher write and read math formulas during their lesson using the math teaching platform. The way of working those all converters to ensure such multimodal interactive formula manipulation by the student and teacher is illustrated in the figure 3. The first four converters from the above list, namely: converters between the MathML, UEB, AsciiMath and the semantic Polish and English mathematical text, were implemented as appropriate JavaScript libraries. The last two converters, namely: Polish BNM to MathML and MathML to BNM, were implemented as appropriate C ++ libraries and sets of configuration files and tables with semantic rules. The open library *Liblouis* was used for their implementation. *Liblouis* is an openly licensed universal conversion library that allows for converting various mathematical notations into Braille, and vice versa [19]. It has been implemented as a C++ module and compiled for a few platforms such as Linux, Unix, Microsoft DotNet, DotNet Core, and Android. Another essential part of this library is a reach set of configuration files, Braille tables and semantic rules, which controls the entire round-trip conversion process, i.e. from MathML to Braille and from Braille to MathML. They are then used for semantic replacement of the symbol sequences from one notation with another. The *Liblouis* library is currently used in many programs supporting the work of the blind, such as screen readers and software for creating Braille publications, and even in software Braille keyboards for mobile phones. *Liblouis* can be adapted to any Braille notation by creating an appropriate set of tables and configuration files and implementing the library's missing functions if needed. Such adaptation to Polish Braille mathematical notation was made as part of our solutions [21]. The converter works by running the translation (MathML Braille) or back-translation (Braille MathML) function. It takes the name of the configuration file for the given Braille notation and the names of the input and output files as arguments. There are references to various tables and semantic files in the configuration file. In these tables, the input characters and the corresponding sequences of characters that will be written to output are encoded. In addition, there are semantic rules that determine in what contexts and in which pass the input sequence is to be converted into an output sequence. The conversion is performed in three following steps.

- In the first step, the input data is initially processed to output. At this stage, some document elements that will be processed in the following stages are appropriately tagged using defined Unicode characters.
- The second pass processes the document more thoroughly. The places marked in the first step are replaced with the appropriate output sign sequences. For example, math formulas are processed using semantic rules stored in the `*.sem` configuration files. Separate rules are forwarding translation (from MathML to Braille) and backward translation (from Braille to MathML).
- The last pass is called the edit pass. This step makes final corrections to the output format, such as adding spaces before the math operators, Braille page numbering, or paragraphs formating.

To bring this process closer, we will give an example of converting a formula from MathML format to Polish Braille mathematical notation. Let take the formula we presented in the previous section, namely:

$$\frac{\frac{1}{2}+\frac{1}{3}}{6}$$

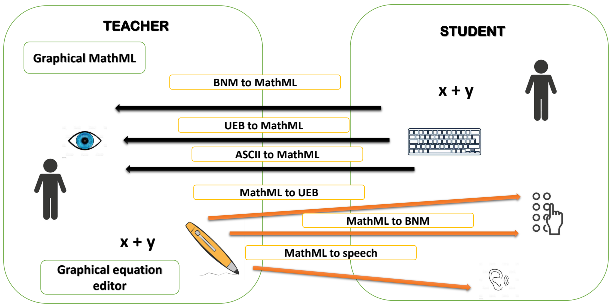


Figure 3: The conversion operations during the manipulation of formulas by the teacher and student

Its MathML representation contains tags such as `<mfrac>`, `<mn>` or `<mo>` to code different math formula parts such as fraction numbers and operators. In the case of the above formula, the converter first states that the input MathML code contains the tag `<frac>`, which means a significant main fraction. Then the conversion rule checks what expressions are in the numerator and denominator of this fraction. It turns out that a fraction has a smaller fraction in the numerator and another fraction in the denominator. Therefore, in the next step, the converter recursively checks what expressions are in the numerators and denominators of these small fractions. Then different MathML to Braille character conversion rules are applied to small fractions and a different rule to the main fraction according to the specification of Braille mathematical notation. The output code contains the fraction beginning sign, spaces and a fraction bar, and the fraction end sign for a significant fraction. A simpler notation is used for small fractions, namely only the sign of the digit and two numbers, one in the numerator and the other in the denominator. The missing spaces are added in the last stage, and the formula is inserted throughout the document. After performing the conversion from this MathML code, we will get the following appropriate ASCII string that denotes the Braille formula.

```
;#a; +#a: 8+<
```

Where: sign `;` (semicolon) means the beginning of the main fraction, `#a;` means small fraction (`#a` means 1 in the numerator) and `;` means 2 in the denominator In addition, signs `#a:` means small fraction $\frac{1}{3}$ and 8 means the fraction bar of the main fraction, and < means the end of the whole fraction. The reverse conversion is also possible, but it is even more difficult because, from the context of the occurrence of Braille symbols, their proximity to the absence of spaces, a neighbourhood of characters, and others, it is necessary to deduce what MathML elements need be generated. After changing the font of the above formula representation sequence to Braille or sending this text to a Braille printer [14, 31], we will get the appropriate pattern of points as shown in the formula presented in the figure 2. As we can note, even from this simple example, coding maths formulas in Braille is not a trivial thing.

4.2 Desktop platform supporting learning math

The converters described above were used in the three proposed solutions for learning mathematics. The first solution we proposed to support mathematics teaching BVI was a desktop platform running under the control of Windows. It enables interactive mathematical communication between blind and visually impaired students and teachers. The platform consists of five desktop applications: an application for a teacher, for a visually impaired student, for a blind student, an advanced calculator and a cubarithm application. All applications can communicate in two ways, namely in the local network or via the Internet. The teacher can create various tasks and worksheets in his application. She/he can write formulas in a graphical editor, create drawings and add text and audio comments to individual elements of the document. These worksheets are then passed on to the students for them to be solved in their applications. After a blind student opens a math document in his application, he can listen to the content of the formula using intelligent semantic text. He can also read them in Braille on his Braille display in the familiar Polish Braille mathematical notation. The learner can also write and modify formulas in Braille using the f d s j k l keys as in a Braille typewriter. Such formulas are converted from BNM to MathMl so that they will then be displayed in the teacher’s application in a graphical form. A visually impaired student can solve a task in his application by enlarging the formula and selecting parts with a different colour.

Another function of the student applications is the so-called formula navigator. Its operation is based on the concept that the formula is represented as a tree. Therefore, the user can move around the tree in all directions and explore its elements. For example, when he has a compound fraction, he first sees two formula elements, namely the numerator and the denominator. He can then expand the numerator and see that there are two more items in it, for example, a sum of expressions. In the next step, he can explore the first expression and learn its content in detail. In this way, he can explore the entire formula and thus understand its structure and the purpose of its elements. The tasks solved by the students are then sent from their applications to the teacher’s application. Then the teacher can check them and give the students further guidance.

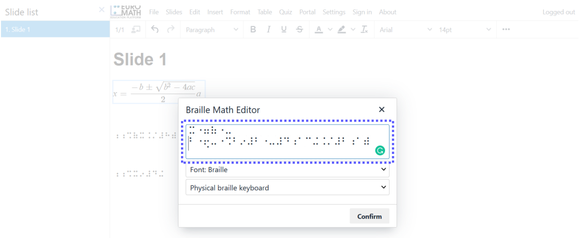
There are also additional applications in the platform, such as cubarithms and a calculator. The calculator works in a standard way, although it is voiced and supplemented with zoom and contrast options so that a BVI student can use it. Cubarithms is a program to perform columnar-layout operations by a BVI student who normally cannot do them on a paper sheet. A blind or visually impaired student can also view drawings using a particular sound graphics navigator. This functionality is described in more detail in our previous publications.

4.3 Web application supporting teaching Math

Our next solution was a web application supporting work with the mathematics of teachers and students with visual problems. It is a web-based system created for students from three countries, Poland, Ireland and the Netherlands. Therefore, students can write mathematical formulas in Braille using Polish mathematical notation and English UEB notation. Unlike the previous system, it is one application in which a teacher, a blind, a visually impaired, and even a sighted student can work with math content. As with the previous solution, a blind student can write a formula in a special Braille editor. In this editor, the f, d. s, j, k, l keys work like the keys on a Braille typewriter. When the student presses them simultaneously, the appropriate Braille character appears on the screen. For example, when he presses the f and d keys simultaneously, a character consisting of points 1 and 2 will be created, i.e. the letter b. Then the student enters the following characters of the formula in the same way and then approves the entire expression. After this step, the mathematical formula entered is converted from Braille to MathML and displayed graphically on the screen so that the teacher can see it. The reverse procedure is also possible. It works as follows. The teacher enters the formula using a graphic editor similar to the one we can meet in Microsoft Word. The formula is then converted from MathML to Braille notation and sent to the student's Braille display. At the same time, it is also converted into a semantic mathematical text and read to the student with a speech synthesizer. A blind student can also load a formula into his Braille editor and modify it. As an example illustrating how this functionality works, we will present a formula for calculating the root of the quadratic equation created by a blind student in the Braille editor of a Web application.

$$x = (-b + -\sqrt{b^2 + -4ac})/2a$$

This formula, loaded in to the Braille editor, is shown in the figure 4.



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}a$$

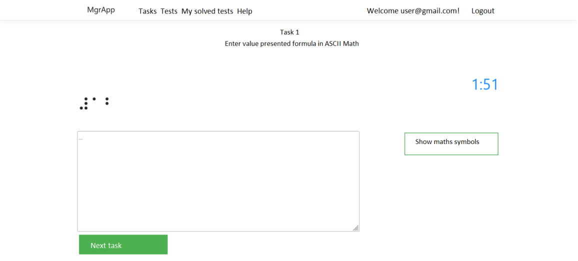
Figure 4: Braille formula in the editor

A blind student can also perform many other operations on the web platform. He can create his graphs and figure drawings by entering parameters such as the function formula or the coordinates of the figure vertices in the appropriate editor. The drawn graph is then played as sounds as the student moves their finger across the touchscreen. It can also be printed on a Braille printer or convex paper.

Another part of the web platform is a portal where teachers can collect worksheets from various fields of mathematics, including them in specially developed categories. The categories have been developed in 3 versions for each country, taking into account specific educational levels and country-specific organization schemas for the education of BVI pupils.

4.4 Application for teachers and tutors

Our last idea is an application for teachers that allows them to learn Braille mathematical notation. It can also be used in the case when a teacher who does not know how to write a given formula in Braille could check it and then prompt the student. The application has been developed as a set of exercises divided into several categories. There are generally two main types of tasks. The first type of exercise is that the user is given formula in asciimath form. His task is to enter the same formula in Braille notation. He can do this by pressing the f, d, s, j, k, l keys representing the Braille dots of each character. Two modes are available here: pressing these keys sequentially or simultaneously. In the first case, entering all points of a given character, e.g. for the letter x, will be points 1, 3, 4, and 6, so the f, s, j, l keys and then press the Braille character separator key. In the second case, he presses these keys simultaneously, and when released, a Braille character appears on the screen. This way, he can type in the entire formula. After its approval, the application converts the formula into MathML and shows it in a graphic form, thanks to which the student can check whether he has entered it correctly in Braille. The second type of exercise is the formula given in Braille notation. The user's task is to recognize it and enter it in the form of AsciiMath notation. If the user is unfamiliar with AsciiMath commands, he can insert them from the toolbar. Then the compliance of the formula is checked, and in case it is correct, the task is solved. The application also has a test mode in which the user has two minutes to solve each task. Then the score is calculated and presented. The figure 5 shows the process of solving the test by a user. As we can see, such an application allows learning of BNM notation by sighted users (teachers, parents and tutors of blind students. It can also help teach maths itself when the teacher needs to prompt the student on how to write a given formula in Braille.



Task 1
Enter value presented formula in ASCII Math

Figure 5: Solving the task in web learning application

5 RESULTS AND DISCUSSION

The solutions for supporting mathematics teaching, i.e. the desktop system and the web application, were tested by students and teachers during mathematics lessons. The Desktop Application was used from 2015-to 2016 by 24 math teachers from 8 schools in Poland. After using the system, quantitative and qualitative surveys were carried out. A more detailed description of them can be found in our previous works [7]. The conclusions from these studies showed great support for the novelty and usefulness of the proposed solutions. The maximum level of support was obtained for the possibility of saving worksheets as universal EPUB3 math documents (20/24 respondents) and the accessible, multimedia, online math resources EPUB3 (20/24 respondents). There were also many uncertain voices in the results regarding the usefulness of some new solutions, e.g. semantic reading of formulas or formula navigation combined with reading. In these cases, the number of respondents supporting the solution and not having an opinion was the same (9/24).

The next stage of the research (in 2017) aimed to determine the measurable benefits of mathematics teachers and BVI due to using our tools in the classroom and at home. After a several-month-long pilot implementation of desktop tools, we have invited five math teachers and eleven students - four blind and seven visually impaired - from 2 special schools for the BVI and one mainstream school. The research was conducted in the form of a questionnaire. The questionnaires had three parts concerning the work of a teacher, the work of a blind pupil and a visually impaired, and the fourth part with the criteria and measures of grades. The main measurable benefits identified were:

- *shortening* the time of performing mathematical operations;
- *increasing* the efficiency of the teacher's service;
- *increasing* the independence of students;
- *reducing* the number of errors made by students;
- *increasing* communication skills;

The blind students found, however, that the time of electronically modifying the formula is the same as the time needed to rewrite the formula traditionally on a Braille typewriter. However, the benefit of using the software is to avoid noise from Braille typewriters in a classroom where, i.e., a group of blind students are working.

The following testing stage was carried out in 2019 after completing work on the web application. It was tested in schools in 3 countries in Poland, Ireland and the Netherlands. Teachers prepared over 300 worksheets for lessons in various fields of mathematics in Polish and English. Then the application itself and the developed worksheets were used in one school in Ireland, one school in the Netherlands and three schools in Poland.

Future users have also pre-verified our third practical solution, namely the application for teaching Polish Braille notation. The application was filled with 38 tasks and two tests. The four sighted and one blind testers makes as many tasks as possible and solve the created tests. After completing this, users were asked to comment on the intuitiveness of the UI and the data input mechanisms used in the application. The testers were to express their satisfaction with the application and the method of entering the formulas on a scale of 1 to 4: 1 not satisfied, 2 - somewhat satisfied, 3 - satisfied and 4 - very satisfied. All testers found that they could learn something new in a relatively pleasant and straightforward way, They recognized that the application has potential power, the interface is clear and legible. They also confirmed that the proposed manner of solving tasks is pleasant, although it requires a few minutes of prior training. Therefore, it is planned to develop it in the future, add even more tasks, and introduce other types of tasks. After these modifications, the use of the application by a larger group of users is planned.

6 CONCLUSIONS

In this paper, we dealt with the problem of the difficulty of teaching mathematics to the BVI, especially to sighted teachers who do not know Braille. We investigated the mathematical Braille notations and IT tools used in different countries. It turns out that all these tools are very partial and focus only on single activities, e.g., Robobrajl is only a tool to convert math documents to Braille, and Desmos helps with learning graphs of functions but does not use Braille at all. The exception is the Lambda system which is a reach math editor available in several languages. Unfortunately, it introduces its notation that students and teachers are reluctant to learn and currently (June 2022), it is not available in Polish. Despite all these technologies, students and teachers still use traditional methods such as paper and Braille typewriters As a solution, we propose techniques for converting MathML to Braille and intelligent semantic speech and vice versa using the best available and most universal converter, Liblouis. Our solution does not introduce new Braille math notation as Lambda does, but it provides tools for the teachers and students who are familiar with BNM and UEB notations. This made it possible to deliver multimodal and interactive access to math content for the student and teacher. In this work, we focused on the formulas and their conversion to Braille and speech, but our solution is much rich. There is an editor for mathematical graphs and figures available for the blind, a system for solving quizzes and electronic cubarithms for solving columnar layout operations and others. We used our method in three developed and verified tools. The most comprehensive of these was a web platform for teaching mathematics to students from three countries: Poland, Ireland and Nederland. However, since the project under which we developed the web platform has been completed, work is underway on its implementation in a new place and will be developed soon. It is all the more important that teachers from three countries participating in the project expressed willingness to continue and expand it. The third latest practical implementation is an application for teachers to learn Polish Braille mathematical notation. It has been verified by several users and received a positive opinion for the idea, but there were many comments about its development, enrichment and supplementation. Therefore, its development and testing among a more significant number of users are planned.

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REFERENCES

[1] F. Alonso and J. Fuertes. 2006. SBT: A Translator from Spanish Mathematical Braille to MathML. In *Proc. of the 10th International Conference on Computers Helping People with Special Needs ICCHP 2006*. Eds. Berlin, Heidelberg, 1207–1214.

[2] D. Andrea. 2013. Unified English Braille: The Future of Braille in the United States. *Journal of visual impairment & blindness* 107 (2013), 243–246.

[3] D. Archambault and F. Guyon. 2011. UMCL transcription tools: Universal Maths Conversion Library. *Assistive Technology Research Series* 29 (2011), 416–423.

[4] D. Archambault, B. Stöger, M. Batusic, C. Fahrengruber, and K. Miesenberger. 2007. A software model to support collaborative mathematical work between Braille and sighted users. In *ASSETS'07: Proceedings of the Ninth International ACM SIGACCESS Conference on Computers and Accessibility*. ACM, 115–122. https://doi.org/10.1145/1296843.1296864

[5] AsciiMath 2014. AsciiMath - Getting Started. http://asciimath.org (Last access: 10 2021).

[6] M. Blomquist. 2002. Braille Contractions in WinBraille. In *Computers Helping People with Special Needs*, Miesenberger K., Klaus J., and Zagler W. (Eds.), Vol. 2398. Springer, Berlin Heidelberg, 602–609. https://doi.org/10.1007/3-540-45491-8-117

[7] J. Brzostek-Pawlowska, M. Rubin, and D Mikulowski. 2019. *Technologie informacyjno-komunikacyjne zwiekszajace dostępność treści matematycznych. in: Adam Marszałek. Toruń (2019).* Adam Marszalek, Torun 20019.

[8] K. Budzanowska and K. Broniarz. 2011. *Brajlowska notacja matematyczna, fizyczna i chemiczna. Edition II, 2011.* Polish Association of the Blind Warszawa (2011).

[9] chezdom.net. 2013. Braille Mathematical Notations. https://chezdom.net/mathematicalbraillecodes/ (last access 03 2022).

[10] L. B. Christensen, S. J. Keegan, and T. Stevns. 2012. SCRIBe: A model for implementing robobraille in a higher education institution. In *Proc. of the 13th International Conference on Computers Helping People with Special Needs, ICCHP 2012*. Eds. Berlin, Heidelberg, 77–83.

[11] Desmos 2017. Inc. Desmos: Desmos graph. https://www.desmos.com/calculator

[12] h. K. Epheser and F. M. Scheid. 1986. *Neufassung und vervollständigung des systems der internationalenmathematikschrift für blinde.* Marburg/L.: StändigeArbeitsgemeinschaft "Mathematikschrift", Deutsche Blindenstudienanstalte.V. 1986. http://www.mediablis-bayern.de/downloads/marburger-mathe-neu-2013.pdf

[13] Euler 2012. Euler - Braille Translation Software. https://www.indexbraille.com/en-us/support/braille-editors/euler (Last access 09 2021).

[14] IndexBraille 1992. Braille Printers / Braille Embossers by Index Braille - Index. https://www.indexbraille.com/ (Last access october 2021).

[15] Italianbraille 2003. Invito al Braille: la matematica. http://www.cavazza.it/invitoalbraille/html/fr32.htm

[16] Miesenberger K. and Batusic i. 2012. MathInBraille online converter. In *Proc. of the 13th International Conference on Computers Helping People with Special Needs, ICCHP 2012*. Eds. Berlin, Heidelberg, 196–203.

[17] C. Kamei-Hannan and Z. Sacks, S. 2012. Parents Perspectives on Braille Literacy: Results from the ABC Braille Study. *Journal of Visual Impairment & Blindness* 106, 4 (2012), 212–223.

[18] Lamport L. 1986. *LaTeX: A Document Preparation System.* Addison-Wesley, Reading, MA.

[19] Liblouis 2003. Liblouis - An open-source braille translator and back-translator. http://liblouis.org/

[20] D. Mikulowski and J. Brzostek-Pawlowska. 2020. Multi-sensual Augmented Reality in Interactive Accessible Math Tutoring System for Flipped Classroom. In *Intelligent Tutoring Systems*. Springer International Publishing, 1–10. https://doi.org/10.1007/978-3-030-49663-0_1

[21] D. Mikulowski and J Brzostek-Pawlowska. 2020. Multi-sensual Augmented Reality in Interactive Accessible Math Tutoring System for Flipped Classroom. In *Intelligent Tutoring Systems*. Springer International Publishing, 1–10. https://doi.org/10.1007/978-3-030-49663-0-1

[22] d. Mikulowski and G. Terlikowski. 2020. Electronic support for developing skills of making columnar layout operations by students with visual impairments. In *EdMedia + Innovate Learning: 2020 Online*. Vol. 1 / 2020. Waynesville, NC Association for the Advancement of Computing in Education (AACE), 1246–1251.

[23] T. Murillo-Morales, K. Miesenberger, and R. Ruemer. 2016. A LaTeX to Braille Conversion Tool for Creating Accessible Schoolbooks in Austria. In *Computers Helping People with Special Needs*, Vol. 9758. Springer, 397–400. https://doi.org/10.1007/978-3-319-41264-1_54

[24] M. Nadeem, N. Aziz, U. Sajjad, F. Aziz, and H. Shaikh. 2016. Comparative analysis of Braille generation technologies. In *International Conference on Advanced Robotics and Mechatronics (ICARM)*. IEEE, 294−−299. https://doi.org/10.1109/ICARM.2016.7606935

[25] Nemeth 1974. Interview with Dr. Abraham Nemeth. https://nfb.org/images/nfb/publications/fr/fr28/fr280110.htm (Last access July 2021).

[26] Royal National Institute of the Blind. 2005. Braille Mathematics Notation. https://chezdom.net\T1\guilsinglrightuploads\T1\guilsinglright2008/07\T1\guilsinglrightbmn

[27] T. V. Raman. 1998. Audio System for Technical Readings. *Lecture Notes in Computer Science* 1410 1998th Edition, 19 (1998), 1–11.

[28] A. Salamonczyk, J. Brzostek-Pawlowska, and D. Mikulowski. 2020. An example of the availability of SVG mathematical graphics on touch screens for the blind supporting remote learning. In *EdMedia + Innovate Learning: 2020 Online*. Vol. 2020, No 1. Waynesville, NC Association for the Advancement of Computing in Education (AACE), 2020, 250–260.

[29] W. Schweikhardt, C. Bernareggi, N. Jessel, B. Encelle, and M. Gut. 2006. LAMBDA: A European system to access mathematics with Braille and audio synthesis. In *International Conference on Computers for Handicapped Persons*. Springer, 1223–1230.

[30] R. D. Stevens, A. D. N. Edwards, W. L. Zagler, G. Busby, and R. L. Wagner. 1994. Mathtalk: The Design of an Interface for Reading Algebra Using Speech. In *Computers for Handicapped Persons: Proceedings of ICCHP '94, Lecture Notes in Computer Science 860*, W L Zagler, G Busby, and R L Wagner (Eds.). Springer Nature, United States, 313–320.

[31] ViewPlus 2018. Take a look at a selection of Viewplus braille printers. https://viewplus.com/braille-printers/ (Last access Apr. 05, 2018).

[32] R. Zatserkovnyi, V. Mayik, R. Zatserkovna, and L. Mayik. 2019. Analysis of Braille translation software. *Printing and Publishing* 2 (2019), 36–44. https://doi.org/10.32403/0554-4866-2019-2-78-36-44

[33] Commission Évolution du Braille Français. 2007. Notation Mathématique Braille. http://chezdom.net/wp-content/uploads/2008/07/notation_mathematique_braille2.pdf